*Fregean Logic (Propositional or Classic Logic)*

The man who first thought a better model was needed was **Gottlob Frege** (we begin to speak about Classic Logic, 19th Century). According to him the logical form of a sentence should not take grammar as its guide, we should rather look towards **mathematics**.

In mathematics there is a particular distinction we should think of, the distinction between **function** and **argument**.

*Function and Argument and Mathematical Analogue*

“Socrates is mortal”

X is mortal = function

Socrates = argument

*Logic as a calculus*

* Propositional logic/ calculus (sentential logic or statement logic)
* Propositions = functions to be filled in by arguments

Frege suggests to change the terminology to describe this sentence. A function is something to be filled in, and what we use to fill it is the argument, after the filling in we have a statement, about which we can say wether it is true or false. What we do is to apply directly the argument to the function. What is the advantage? The advantage is that it allows us to think in much more general and fungible way.

Mathematical analogue:

“the square of (x) = y” “the capital of (x) = y” The same mechanism applies to any filling, and we’ll get a particular result that matches. We can now represent different relations and properties with the same functional notations. I can say of each of them if I’m reasoning in term of truthiness or falseness.

Logic becomes a kind of calculus. That’s why after Frege, logic is also known as Propositional calculus. Propositions are functions to be filled in.

*Difference with categorical logic*

There are some differences with categorical logic.

* “All humans are mortal”  H. and M. stand for categories, classes of object in Aristotelian logic. In Frege’s they stand for the whole proposition (it is the whole proposition which has truth value).
* Mathematics is used as a model instead of grammar
* Statements are functions to be filled in by arguments, this includes a wider category than syllogism.

*A look inside proposition (predicate logic)*

We have a much more general way of dealing with statements. We need now to study the relation between propositions, through the use of a new apparatus:

* **Connectives**:  Given two statements, they can be connected in various way and the outcome is a statement which truth value is given by the connection (This is why they are also called truth functions). The most frequently used are:  - negation :  Given a statement p, we can negate that statement. So “not p”, and say that it can be either true or false (both the positive and negative).  We can then represent graphically this situation through a ‘Truth Table’. These table help us visualize some of the principles of Aristotelian logic. (Principle of non contradiction: the table tell us that it is impossible to state the trueness and the falseness of a statement in the same time.)  - conjunction - disjunction - material implication - double material implication
* **Quantifiers**:  “All men are mortal” “Every man is mortal” “Some man is bold”  all, every, some are examples of quantifiers.  It is their presence in each statement that makes the statement a ‘Quantifier Statement’. The quantifiers “all” and “every” are universal quantifiers (Symbol ∀). Moreover there are existential quantifiers like “some” which are represented by the symbol ∃.  Given that in a statement we predicate something about something else, we get either: - Quantified universal statements: All x are f; For every x, x is an f  - Quantified existential statements: There is an x that is f; Some x is f. There is always an x related to a quantifier.
* **Variables**: A symbol which is used to refer to something else that is not a symbol.  It is a Place Holder/ Notation, where some kind of a substitution can take place.  A variable is different from a constant, that has a fixed meaning; a variable can be filled with a different array of things. They can be - Bound, if their occurrence is completely guided by a quantifiers. - Free, not so bound by a quantifier.

*Complex Quantified Elements*

**By putting statements together we get a complex statement, the truth value of which depends on the trueness of the basic statements**.  “All dogs are mammals”.

*“All Men”*

* In the Aristotelian case the quantifier ‘all’ can be only applied to a well defined domain, which is the subject of the statement. ‘All men’ applies to the domain ‘human beings’.
* In the Fregean sense: ‘For every x, in those cases where x is a man’. We translate the sentence ‘For every x, if x is a man x is mortal’. (**More applicability)** It is a **conditional reasoning**. The **truth value** of the sentence is independent of categories, classes of object, and can be calculated over and over again, independently of its content (**independent from classes and content**). What counts is the inferential form, that can be transferred to one context to another and still be working (generality as an advantage).

**What is the meaning of the copula ‘is’?**

By means of Frege we can figure out all the possibilities of meaning of the copula itself: “Socrates is Xanthippe’s husband” -> identity, a=b

“Socrates is a man” -> the copula expresses the belonging to a general domain

“There are men” -> existential sense of the verb (if there are x, I pick the x that are men) “All men are mortal” -> inclusive manner, we include two domains together.

**RECAP**

Two types of logic that share some features and diverge on others:

**Categorical logic (Aristotle)**: a logic that includes the representation of classes of things and look at the grammatical relation between them.

* Logic of classes or categories
* Logic of statements that can be represented in terms of classes of things, and relationships between those classes
* Logic that gives us tools for representing categorical statements of the type: All S are P; Some S are P No S are P; Some S are not-P
* All the statements respect the grammatical form Subject + Copula.
* It gives us logical tools
* It is a logic of syllogism, for which the model of correct reasoning is the syllogism. It is an argument consisting of Three categorical statements, two premises and a conclusion.
* Three categorical term appears, each of them used twice.
* It is a logic that wants to figure out what correct reasoning is, to be able to build a valid argument. If an argument doesn’t follow these rules, it will be called invalid, a fallacy.
* It is a binary logic, either be true or false.
* It is a logic that separates valid arguments from invalid arguments (fallacies)

**Propositional logic (Frege)**: deals with logical relationship between statements taken as wholes.

* The truth value of the statement deals with the entire proposition.
* Simple propositions can be combined to make more complex propositions (which can themselves ve treated as single propositions with their own truth
* What determines the truth value of the whole sentence is the truth values of the individual component sentence, together with the rules for interpreting logical connectives like “and”, “or” and “if …then…”
* In propositional logic we learn the rules for determining the truth values of the following compound claims: not-p, p and q, p or q, if p then q.
* It can handle all combinations of quantifiers
* It can handle conjunctions, disjunctions, conditionals and biconditionals
* It can represent predicates that involve relations between variables

**Predicate Logic (Aristotle and Frege)**:

* It formalises the different parts of a proposition
* It can handle all the combinations of quantifiers
* It includes the whole of categorical logic, but also.. Fregean Logic
* It can handle conjunctions, disjunctions, conditionals and biconditionals
* It can represent predicates that involve relations between variables

Comparison:

To some extent, both types of logic shares some features: Both are binary; they are both types of predicate logic, because they look at what we predicate in a statement; they pay attention to the parts that make up the proposition, although they look at different parts.

Fregean logic has some advantages: it can handle all combination of quantifiers, and it can also represent predicated that involve relations between variable.

Not ALL logic are of these types. There are other types of logic, which do not assume that propositions can only be true or false, that there are not any other way.